5.2 Blasius theorem:- (Statement)

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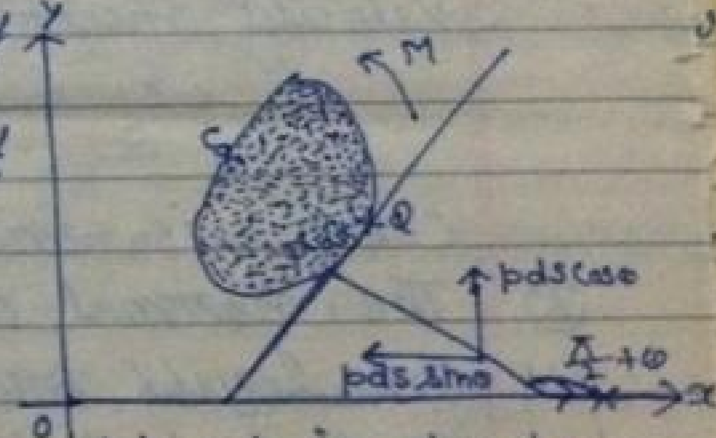
I Consider a fixed cylinder is moving steady and irrotationally of an incompressible fluid under the action of no extraneous force by the complex Potential $w = f(z)$ if the hydrodynamical Pressure on the control of a fixed cylinder are represent by a force (X, Y) and a couple M about the origin of cordinates, then

$$X - iY = \frac{1}{2} i\rho \int_c \left(\frac{dw}{dz}\right)^2 dz$$

And $$M = \frac{1}{2}\rho\,(R.P\,of) \int_c z \left(\frac{dw}{dz}\right)^2 dz \qquad \text{I}$$

II **Proof:-** The flow is steady and irrotational flow of a fluid that velocity Potential ϕ exist.



Consider a cylinder of any cross-section placed in steady. Let $P(x,y)$ be any Point on the cylinder on an element δs. the tangent at any Point $P(x,y)$ makes an angle θ with x-axis. the fluid thrust at P will act along the inward drawn normal to the cylinder. the elemental force component on P. along x axis and y-axis are given as:

$$dX = p\,ds\cos(\theta + \pi/2) = -p\,ds\sin\theta$$

$$dY = p\,ds\sin(\theta + \pi/2) = p\,ds\cos\theta$$

Where p is the Pressure on the cylinder. II

$$\frac{1}{r}\frac{d}{dr}\left(r f' \cos\theta\right) - \frac{1}{r^2}\cos\theta = 0$$

$$\Rightarrow \frac{1}{r}\left[r f'' + f'\right]\cos\theta - \frac{f}{r^2}\cos\theta = 0$$

$$\Rightarrow \left[f'' + \frac{1}{r} f' - \frac{1}{r^2} f\right]\cos\theta = 0$$

$$(r^2 f'' + r f' - f)\cos\theta = 0$$

$$\cos\theta \neq 0 \Rightarrow \theta = \pi/2$$

$$r^2 \frac{d^2 f}{dr^2} + r\frac{df}{dr} - f = 0$$

$$\boxed{\begin{gathered}\{D(D-1) + D - 1\} f = 0 \\ m(m-1) + m - 1 = 0 \\ m^2 - 1 = 0 \Rightarrow m = \pm 1\end{gathered}}$$

$$f(r) = Ar + \frac{B}{r}$$

$$\phi = \left(Ar + \frac{B}{r}\right)\cos\theta$$

$$\frac{\partial\phi}{\partial r} = \left(A - \frac{B}{r^2}\right)\cos\theta$$

$$-\frac{\partial\phi}{\partial r} \to 0 \quad \text{As } r \to 0$$

$$A\cos\theta = 0 \Rightarrow A = 0$$

$$\Rightarrow -U\cos\theta = -\frac{B}{a^2}\cos\theta \Rightarrow B = Ua^2$$

$$\boxed{\phi = \frac{Ua^2}{r}\cos\theta}$$

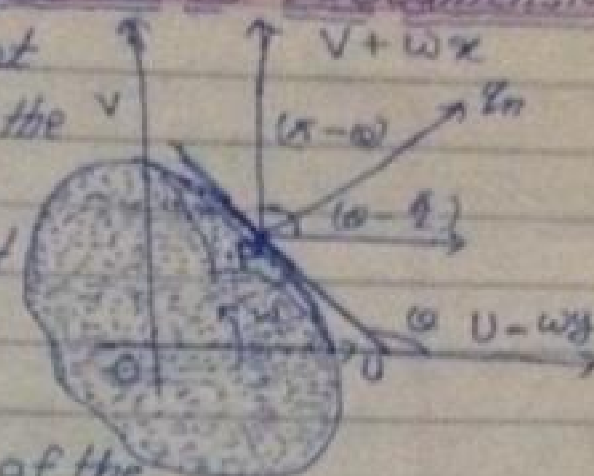
$$\psi = -\frac{Ua^2}{r}\sin\theta$$

Hence the complex Potential

$$w = \phi + i\psi = \frac{Ua^2}{r}(\cos\theta - i\sin\theta)$$

$\frac{5.17}{236}$:- General Motion of a cylinder in two dimension

Let $P(x, y)$ be any Point on the boundary of the cylinder and the tangenth at the Point P makes an angle θ with x axis



Let U And V Velocity of the cylinder Parallel to the axis of x and y. Let ω be the Angular Velocity component at the Point P are given to Along x axis

$$U + \frac{dx}{dt}$$

Along y axis $\quad V + \frac{dy}{dt}$

$$x = r\cos\theta \Rightarrow \frac{dx}{dt} = -r\sin\theta\frac{d\theta}{dt} = -y\omega$$

$\frac{d\theta}{dt} = \omega$

$$y = r\sin\theta \Rightarrow \frac{dy}{dt} = r\cos\theta\frac{d\theta}{dt} = x\omega$$

The outward normal velocity

$$= (U-\omega y)\cos\left(\theta-\frac{\pi}{2}\right) + (V+\omega x)\cos(\pi-\theta)$$

$$= (U-\omega y)\sin\theta - (V+\omega x)\cos\theta$$

The Normal velocity

$$-\frac{\partial\psi}{\partial s} = (U-\omega y)\frac{dy}{ds} - (V+\omega x)\frac{dx}{ds}$$

$$= (U\,dy - V\,dx) - \omega(x\,dx + y\,dy)$$

$$\psi = (Vx - Uy) + \frac{1}{2}\omega(x^2+y^2) + c \quad \text{--- (1)}$$

Which determines the current function for the most General type of motion of the circular cylinder

I :- If there irrotation then

$$\omega = 0,\ V = 0$$

the current function reduces to

$$\boxed{\psi = c - Uy} \quad \text{--- (2)}$$

The net force acting on the element PQ

$$dF = dx + i\,dy$$

$-\frac{1}{i}$

$$= -p\,ds\sin\theta + i p\,ds\cos\theta \quad ip\,ds(\cos\theta + i\sin\theta)$$

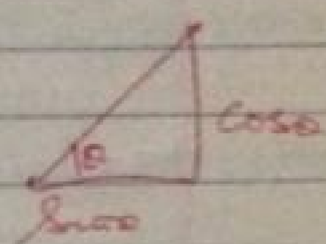
$-\frac{i}{i^2}$

$$= -ip\,ds\sin\theta + ip\,ds\cos\theta = ip\,e^{\theta i}ds$$

$$dF = ip\,ds(\sin\theta + i\sin\theta)$$

Now $dz = dx + i\,dy$

$$= \left(\frac{dx}{ds} + i\frac{dy}{ds}\right)ds$$



$$= (\cos\theta + i\sin\theta)ds$$

$$\boxed{dz = e^{\theta i}ds}$$

$$dF = ip\,e^{\theta i}ds$$

$$\boxed{dF = ip\,dz} \quad \text{--- (1)}$$

The Pressure in a steady irrotational flow By Bernoulli's theorem

$$p = p_0 - \frac{1}{2}\rho q^2 \quad \text{--- (2)}$$

where p_0 is the Pressure at the stagnation is the speed at the Point P

the complex Potential is given by the relation

$$w = \phi + i\psi = f(x+iy)$$

$$\frac{dw}{dx} = \frac{\partial\phi}{\partial x} + i\frac{\partial\psi}{\partial x}$$

$$\frac{dw}{dz}\cdot\frac{dz}{dx} = \frac{\partial\phi}{\partial x} + i\frac{\partial\psi}{\partial x}$$

$$\frac{dw}{dz} = \frac{\partial\phi}{\partial x} - i\frac{\partial\phi}{\partial y}$$

$$\frac{dw}{dz} = -u + iv$$

$$\frac{dw}{dz} = -q(\cos\theta - i\sin\theta)$$

$$= -q\,e^{-\theta i} \quad \text{--- (3)}$$

Int. (1) we have

$$\Rightarrow \int_c dF = \int_c ip\,dz$$

$$= \frac{Ua^2}{r} e^{-\theta i}$$

$$w = \frac{Ua^2}{re^{\theta i}} = \frac{Ua^2}{z}$$

Then stream line are given by

$$\psi = \text{const.}$$

$$\frac{Ua^2}{r} \sin\theta = c \quad \text{where } c \text{ is constant}$$

$$\sin\theta = \frac{cr}{Ua^2}$$

$$Ua^2 \sin\theta = cr \Rightarrow Ua^2 r \sin\theta = cr^2$$

or $$\boxed{x^2+y^2-ky=0}$$

which is represent the circle all touching x-axis at origin.

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